

# Massless solutions for families of spinors for the toy model [1] in $d = (1 + 5)$

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Massless solutions for four coupled first order differential equations for functions representing families of spinors on the infinite disc curled into an almost  $S^2$  are presented and normalizability conditions discussed. Spinors interact with two kinds of spin connection fields of particular coordinate dependence [1].

## I. INTRODUCTION

We present in this contribution massless solutions of equations of motion for families of spinors on an infinite disc curled on an almost  $S^2$ . Spinors interact with two kinds of the spin connection fields. The action leads [1] to two decoupled groups of four coupled first order differential equations, each for four functions. The two groups are not really unconnected: There are three spin-connection fields which appear in both groups of four equations. The strengths of these and other spin connection fields determine possible massless solutions.

Below four coupled first order differential equations for one of the two groups are presented [1]

$$\begin{aligned}
& -if \left\{ \left[ \left( \frac{\partial}{\partial \rho} - \frac{n}{\rho} \right) - \frac{1}{2f} \frac{\partial f}{\partial \rho} (1 - 2F_{56} - 2\tilde{F}_{56} - 2\tilde{F}^{\ominus 3}) \right] \mathcal{A}_n^I \right. \\
& - \frac{1}{2f} \frac{\partial f}{\partial \rho} 2\tilde{F}^{\ominus \oplus} \mathcal{A}_n^{II} \left. \right\} + m \mathcal{B}_{n+1}^I = 0, \\
& -if \left\{ \left[ \left( \frac{\partial}{\partial \rho} + \frac{n+1}{\rho} \right) - \frac{1}{2f} \frac{\partial f}{\partial \rho} (1 + 2F_{56} - 2\tilde{F}_{56} - 2\tilde{F}^{\ominus 3}) \right] \mathcal{B}_{n+1}^I \right. \\
& - \frac{1}{2f} \frac{\partial f}{\partial \rho} 2\tilde{F}^{\ominus \oplus} \mathcal{B}_{n+1}^{II} \left. \right\} + m \mathcal{A}_n^I = 0,
\end{aligned} \tag{1}$$

$$\begin{aligned}
& -if \left\{ \left[ \left( \frac{\partial}{\partial \rho} - \frac{n}{\rho} \right) - \frac{1}{2f} \frac{\partial f}{\partial \rho} (1 - 2F_{56} - 2\tilde{F}_{56} + 2\tilde{F}^{\ominus 3}) \right] \mathcal{A}_n^{II} \right. \\
& - \frac{1}{2f} \frac{\partial f}{\partial \rho} 2\tilde{F}^{\ominus \oplus} \mathcal{A}_n^I \left. \right\} + m \mathcal{B}_{n+1}^{II} = 0, \\
& -if \left\{ \left[ \left( \frac{\partial}{\partial \rho} + \frac{n+1}{\rho} \right) - \frac{1}{2f} \frac{\partial f}{\partial \rho} (1 + 2F_{56} - 2\tilde{F}_{56} + 2\tilde{F}^{\ominus 3}) \right] \mathcal{B}_{n+1}^{II} \right. \\
& - \frac{1}{2f} \frac{\partial f}{\partial \rho} 2\tilde{F}^{\ominus \oplus} \mathcal{B}_{n+1}^I \left. \right\} + m \mathcal{A}_n^{II} = 0.
\end{aligned}$$

The parameters  $(F_{56}, \tilde{F}_{56}, \tilde{F}^{\ominus \oplus}, \tilde{F}^{\ominus \ominus}, \tilde{F}^{\ominus 3})$  are assumed to be free.

For the massless case these four coupled equations decouple into equations for  $\mathcal{A}_n^{I,II}$

$$\begin{aligned} [(\frac{\partial}{\partial \rho} - \frac{n}{\rho}) - \frac{1}{2f} \frac{\partial f}{\partial \rho} (1 - 2F_{56} - 2\tilde{F}_{56} - 2\tilde{F}^{\ominus 3})] \mathcal{A}_n^I - \frac{1}{2f} \frac{\partial f}{\partial \rho} 2\tilde{F}^{\ominus \oplus} \mathcal{A}_n^{II} &= 0, \\ [(\frac{\partial}{\partial \rho} - \frac{n}{\rho}) - \frac{1}{2f} \frac{\partial f}{\partial \rho} (1 - 2F_{56} - 2\tilde{F}_{56} + 2\tilde{F}^{\ominus 3})] \mathcal{A}_n^{II} - \frac{1}{2f} \frac{\partial f}{\partial \rho} 2\tilde{F}^{\ominus \oplus} \mathcal{A}_n^I &= 0, \end{aligned} \quad (2)$$

and into equations for  $\mathcal{B}_{n+1}^{I,II}$

$$\begin{aligned} [(\frac{\partial}{\partial \rho} + \frac{n+1}{\rho}) - \frac{1}{2f} \frac{\partial f}{\partial \rho} (1 + 2F_{56} - 2\tilde{F}_{56} - 2\tilde{F}^{\ominus 3})] \mathcal{B}_{n+1}^I - \frac{1}{2f} \frac{\partial f}{\partial \rho} 2\tilde{F}^{\ominus \oplus} \mathcal{B}_{n+1}^{II} &= 0, \\ [(\frac{\partial}{\partial \rho} + \frac{n+1}{\rho}) - \frac{1}{2f} \frac{\partial f}{\partial \rho} (1 + 2F_{56} - 2\tilde{F}_{56} + 2\tilde{F}^{\ominus 3})] \mathcal{B}_{n+1}^{II} - \frac{1}{2f} \frac{\partial f}{\partial \rho} 2\tilde{F}^{\ominus \oplus} \mathcal{B}_{n+1}^I &= 0. \end{aligned} \quad (3)$$

Both groups of equations can be solved in an equivalent way.

Let us first find solutions for  $\mathcal{A}_n^{I,II}$ . In the case that  $\tilde{F}^{\ominus \oplus} \tilde{F}^{\ominus \ominus} = 0 = \tilde{F}^{\ominus 3}$  the two functions decouple and one easily finds the solution [2, 3]:  $\mathcal{A}_n^{I,II} = \mathcal{N} \rho^n f^{\frac{1}{2}(1-2F_{56}-2\tilde{F}_{56})}$ . In our case we correspondingly try with the following ansatz

$$\begin{aligned} \mathcal{A}_n^I &= \rho^n f^{\frac{1}{2}(1-2F_{56}-2\tilde{F}_{56})} [a_1 f^\alpha + b_1 f^\beta], \\ \mathcal{A}_n^{II} &= \rho^n f^{\frac{1}{2}(1-2F_{56}-2\tilde{F}_{56})} [a_2 f^\alpha + b_2 f^\beta], \end{aligned} \quad (4)$$

When using this ansatz in Eq. (2) we end up with the relations

$$\begin{aligned} f^\alpha (-a_1 \tilde{F}^{\ominus 3} - \alpha a_1 + a_2 \tilde{F}^{\ominus \oplus}) &= 0 = f^\beta (-b_1 \tilde{F}^{\ominus 3} - \beta b_1 + b_2 \tilde{F}^{\ominus \oplus}), \\ f^\alpha (a_2 \tilde{F}^{\ominus 3} - \alpha a_2 + a_1 \tilde{F}^{\ominus \oplus}) &= 0 = f^\beta (b_2 \tilde{F}^{\ominus 3} - \beta b_2 + b_1 \tilde{F}^{\ominus \oplus}). \end{aligned} \quad (5)$$

We correspondingly find

$$\begin{aligned} a_2 &= a_1 \frac{\alpha + \tilde{F}^{\ominus 3}}{\tilde{F}^{\ominus \oplus}} = a_1 \frac{\tilde{F}^{\ominus \oplus}}{\alpha - \tilde{F}^{\ominus 3}}, \\ \alpha &= \pm \sqrt{(\tilde{F}^{\ominus 3})^2 + \tilde{F}^{\ominus \oplus} \tilde{F}^{\ominus \ominus}}, \\ b_2 &= b_1 \frac{\beta + \tilde{F}^{\ominus 3}}{\tilde{F}^{\ominus \oplus}} = b_1 \frac{\tilde{F}^{\ominus \oplus}}{\beta - \tilde{F}^{\ominus 3}}, \\ \beta &= \pm \sqrt{(\tilde{F}^{\ominus 3})^2 + \tilde{F}^{\ominus \oplus} \tilde{F}^{\ominus \ominus}} = \alpha. \end{aligned} \quad (6)$$

General massless solutions of Eq. (2) are then

$$\begin{aligned} \mathcal{A}_n^I &= a \rho^n f^{\frac{1}{2}(1-2F_{56}-2\tilde{F}_{56})} f^\alpha, \\ \mathcal{A}_n^{II} &= a \frac{\alpha + \tilde{F}^{\ominus 3}}{\tilde{F}^{\ominus \oplus}} \rho^n f^{\frac{1}{2}(1-2F_{56}-2\tilde{F}_{56})} f^\alpha, \end{aligned} \quad (7)$$

with  $\alpha$  equal to either  $-\sqrt{(\tilde{F}^{\ominus 3})^2 + \tilde{F}^{\ominus \oplus} \tilde{F}^{\ominus \ominus}}$  or to  $+\sqrt{(\tilde{F}^{\ominus 3})^2 + \tilde{F}^{\ominus \oplus} \tilde{F}^{\ominus \ominus}}$ . One can take also a superposition of both, with related  $\mathcal{A}_n^I$  and  $\mathcal{A}_n^{II}$  as required by Eqs. (6, 7).

In a similar way we get massless solutions of Eq. (3)

$$\begin{aligned}\mathcal{B}_{n+1}^I &= b \rho^{-n-1} f^{\frac{1}{2}(1+2F_{56}-2\tilde{F}_{56})} f^\alpha, \\ \mathcal{B}_{n+1}^{II} &= b \frac{\alpha + \tilde{F}^{\ominus 3}}{\tilde{F}^{\ominus \boxplus}} \rho^{-n-1} f^{\frac{1}{2}(1+2F_{56}-2\tilde{F}_{56})} f^\alpha,\end{aligned}\tag{8}$$

again with  $\alpha = \mp \sqrt{(\tilde{F}^{\ominus 3})^2 + \tilde{F}^{\ominus \boxplus} \tilde{F}^{\ominus \boxminus}}$ .

A function  $\mathcal{G}$ , defined on an infinite disc with the vielbein [2]  $f = (1 + \frac{\rho^2}{(2\rho_0)^2})$ , is normalizable provided

$$\begin{aligned}\int_0^\infty \rho d\rho f^{-2} |\mathcal{A}_n^{I,II}|^2 &< \infty, \\ \int_0^\infty \rho d\rho f^{-2} |\mathcal{B}_{n+1}^{I,II}|^2 &< \infty.\end{aligned}\tag{9}$$

Correspondingly are the solutions  $\mathcal{A}_n^{I,II}$  normalizable under the condition

$$-1 < n < 2(F_{56} + \tilde{F}_{56} \pm \sqrt{(\tilde{F}^{\ominus 3})^2 + \tilde{F}^{\ominus \boxplus} \tilde{F}^{\ominus \boxminus}}).\tag{10}$$

The equivalent condition follows from normalizability requirement for  $\mathcal{B}_n^{I,II}$

$$2(F_{56} - \tilde{F}_{56} \pm \sqrt{(\tilde{F}^{\ominus 3})^2 + \tilde{F}^{\ominus \boxplus} \tilde{F}^{\ominus \boxminus}}) < n < 1.\tag{11}$$

To find massive solutions one could follow the procedure of the ref. [4].

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